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# A complete solution to neutrino mixing

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## Abstract

Deviations from expectations have been claimed for solar, atmospheric and high energy prompt neutrinos from charm decay. This information, supplemented only by the existing very good upper limits for oscillations of the  $\nu_\mu$  at accelerator energies, is used as input to a phenomenological three-flavour analysis of neutrino mixing. The solution found is unique and completely determines the mass eigenstates as well as the mixing matrix relating mass and flavour eigenstates. Assuming the mass eigenstates to follow the hierarchy  $m_1 \ll m_2 \ll m_3$ , their values are found to be  $m_1 \ll 10^{-2}$  eV,  $m_2 = (0.18 \pm 0.06)$  eV,  $m_3 = (19.4 \pm 0.7)$  eV. These masses are in agreement with the leptonic quadratic hierarchy of the see-saw model and large enough to render energy-independent any oscillation-induced phenomenon in solar neutrino physics observable on Earth. This possibility is not excluded by the present knowledge of solar neutrino physics. The mixing angles are determined to be  $\theta_{12} = 0.55 \pm 0.08$ ,  $\theta_{13} = 0.38 \pm 0.06$ ,  $\theta_{23} < 0.03$ . Small values of  $\theta_{23}$  are typical of any solution in which  $m_3$  lies in the cosmological interesting region. The solution found is not in serious contradiction with any of the present limits to the existence of neutrino oscillations. The most relevant implications in particle physics, astrophysics and cosmology are discussed.

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# 1 Introduction

In recent years, several deviations from expectations have been reported in various fields of neutrino physics [1].

At high energy, the equality between the  $\nu_e$  and  $\nu_\mu$  energy spectra prescribed by e- $\mu$  universality for the semi-leptonic decays of charmed particles appears to be violated at the four sigma level.

In atmospheric neutrinos, the ratio between the  $\nu_\mu$  and  $\nu_e$  fluxes is found to be substantially less than the predicted value of 2.

Even if quantitative conclusions depend on the somewhat different predictions of the various solar models, measurements of solar neutrino fluxes on Earth have been known for some time to fall short of the corresponding theoretical expectations.

These positive results have been taken as indications of the existence of non-zero neutrino masses and mixings. However, they have to be examined for consistency with each other and confronted with the many negative searches for oscillations and their corresponding upper limits.

No satisfactory purely phenomenological over-all interpretation has been obtained so far.

In this paper we describe a comprehensive three-flavour analysis which results in an unique, overconstrained and complete description of neutrino mixing. Negative results being usually available only at the 1.64 sigma level (90% CL), we have used as input data only the three positive results above and the two most stringent upper limits obtained in the searches for  $\nu_e$ - $\nu_\mu$  and  $\nu_\mu$ - $\nu_\tau$  oscillations at high energy accelerators. We also discuss the compatibility of this solution with the other existing limits and its most relevant implications in particle physics and astrophysics.

Preliminary accounts of this work have been presented elsewhere [2, 3].

CP-invariance is assumed throughout the paper. Also, unless otherwise stated, the term neutrino is used to indicate both neutrino and antineutrino.

## 2 Three-flavour formalism

In the complete three-flavour approach, the weak eigenstates  $|\nu_\alpha\rangle = \nu_e, \nu_\mu, \nu_\tau$  and the mass eigenstates  $|\nu_i\rangle = \nu_1, \nu_2, \nu_3$  are related by an unitary transformation matrix  $U$ , in terms of which the probability of an initial neutrino  $\nu_\alpha$  of energy  $E$  being equal to another neutrino  $\nu_\beta$  at a distance  $L$ , can be written as

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2(\Delta_{ij}/2) \quad (1)$$

with  $\Delta_{ij} = \delta m_{ij}^2 L/2E$ , where  $\delta m_{ij}^2 = m_i^2 - m_j^2$ ,  $m_i = m(\nu_i)$ .

The  $U$ -matrix can be parametrized as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} \quad (2)$$

with  $c_{ij} = \cos \theta_{ij}$  and  $s_{ij} = \sin \theta_{ij}$ , where  $\theta_{12}, \theta_{13}$  and  $\theta_{23}$  are three independent real angles lying in the first quadrant.

Of the three  $\delta m_{ij}^2$ 's appearing in eq. (1), only two are independent. The complete solution of the problem consists therefore in determining five unknowns: two  $\delta m_{ij}^2$ 's and the three  $\theta_{ij}$ 's.

For  $m_1 \ll m_2 \ll m_3$ , only two  $\delta m_{ij}^2$ 's characterize the oscillatory behaviour of eq. (1):  $\delta m_{12}^2$  and  $\delta m_{13}^2 \simeq \delta m_{23}^2 \gg \delta m_{12}^2$ , corresponding, respectively, to a “slow” and a “fast” oscillation. Thus, for any given experimental situation, depending on the range of  $L/E$  under study, either only the fast or both the fast and slow oscillations may be occurring.

This allows to define the two ranges  $1/\delta m_{13}^2(\text{eV}^{-2}) \ll L/E(\text{m/MeV}) \ll 1/\delta m_{12}^2(\text{eV}^{-2})$  (“short-baseline”) and  $L/E(\text{m/MeV}) \gg 1/\delta m_{12}^2(\text{eV}^{-2})$  (“long-baseline”) in which the average transition probabilities  $P_{\alpha\beta}^S$  and  $P_{\alpha\beta}^L$  are calculated from eq. (1) for  $\sin^2(\Delta_{12}/2) = 0$ ,  $\langle \sin^2(\Delta_{13}/2) \rangle = 0.5$  and  $\langle \sin^2(\Delta_{12}/2) \rangle = 0.5$ ,  $\langle \sin^2(\Delta_{13}/2) \rangle = 0.5$ , respectively.

## 3 Input data

### 3.1 Accelerator neutrinos

In prompt neutrinos from charm decay, the equality between the  $\nu_e$  and  $\nu_\mu$  spectra prescribed by e- $\mu$  universality appears to be violated as the neutrino flux asymmetry

$$A = (\nu_\mu \text{ flux} - \nu_e \text{ flux})/(\nu_\mu \text{ flux} + \nu_e \text{ flux}) \quad (3)$$

is experimentally determined to be  $A = 0.21 \pm 0.05$  [4].

For values of  $L/E$  smaller than about 0.1 m/MeV, and consequently in the domain of the prompt neutrino data,  $P_{e\mu}$  and  $P_{\mu\tau}$  are experimentally known to obey the 90% CL upper limits [5]

$$P_{e\mu} < 1.5 \times 10^{-3}, \quad P_{\mu\tau} < 2 \times 10^{-3}. \quad (4)$$

Thus, ascribing the non-vanishing value of  $A$  to a depletion of the  $\nu_e$  flux due to oscillations, the analysis of the  $L/E$  dependence of the data leads to the existence of  $\nu_e$ - $\nu_\tau$  oscillations with the parameters [4, 6]

$$\sin^2(2\alpha) = 0.48 \pm 0.10 \pm 0.05 \quad (5)$$

$$\delta m^2 = (377 \pm 27 \pm 7) \text{ eV}^2. \quad (6)$$

Table 1: Atmospheric neutrino experimental results.

EXPERIMENT	$r = (\mu/e)_{\text{obs}}/(\mu/e)_{\text{calc}}$
Nusex	$1.04 \pm 0.32$
Frejus (contained events)	$0.87 \pm 0.19$
IMB-3	$0.54 \pm 0.13$
Kamiokande	$0.60 \pm 0.08$
Soudan 2	$0.64 \pm 0.19$

In terms of three-flavour notations, eq. (6) ensures this to be the fast oscillation ( $\sin^2(\Delta_{12}/2) = 0$ ), and the limits of eq. (4) imply that  $\theta_{23}$  must be very small. It then follows that eqs. (5) and (6) can be rewritten as

$$\sin^2(2\theta_{13}) = 0.48 \pm 0.12 \quad (7)$$

$$\delta m_{13}^2 \simeq \delta m_{23}^2 = (377 \pm 29) \text{ eV}^2. \quad (8)$$

### 3.2 Atmospheric neutrinos

Neutrinos are also produced in the interactions of the primary component of cosmic rays in the Earth's atmosphere and in the subsequent decays of the produced secondaries. As almost all these decays involve a muon, which in turn also decays, the ratio  $R$  between the  $\nu_\mu$  and  $\nu_e$  fluxes can be safely predicted to be approximately 2.

Several experiments have addressed this question [7]. They all consistently find values of  $R$  which are smaller than 2 by a factor of  $r$ , approximately equal to 0.6. In fact, for average neutrino energies below 1 GeV, the weighted average of all available data [8, 9, 10, 11, 12] listed in Table 1 gives

$$r = 0.63 \pm 0.06 \quad \text{with } \chi^2/\text{d.o.f.} = 0.96. \quad (9)$$

Contrary to the low-energy behaviour, in the multi-GeV energy range  $r$  is observed to be a function of the zenith-angle[13]. This is very suggestive of an  $L/E$  dependence of  $r$  and thus of an oscillation-induced phenomenon. The  $\delta m^2$  characterizing this oscillation lies in the 90% CL interval

$$5 \times 10^{-3} < \delta m_{12}^2 < 8 \times 10^{-2} \text{ eV}^2. \quad (10)$$

Eq. (10) ensures that in the sub-GeV energy range all detected observables have their average long-baseline values. Thus, in terms of the three-flavour formalism, eq. (9) becomes

$$(P_{\mu\mu}^L + \rho P_{e\mu}^L)(\rho^{-1} P_{e\mu}^L + P_{ee}^L)^{-1} = 0.63 \pm 0.06 \quad (11)$$

where  $\rho = 0.47 \pm 0.02$  is the expected  $\nu_e/\nu_\mu$  flux ratio in the absence of oscillations [14, 15]

Table 2: Solar neutrino experimental results and theoretical predictions ( $1\sigma$ ). Statistical and systematic errors have been added in quadrature. For the  $\text{Ga}^{71}$  entry, the quoted experimental value is the weighted average of the SAGE ( $69 \pm 10^{+5}_{-7}$  SNU [18]) and Gallex ( $77.1 \pm 8.5^{+4.4}_{-5.4}$  SNU [19]) results. The error on this value is almost certainly underestimated as systematic errors have been treated as uncorrelated.

	DETECTOR		
	$\text{Cl}^{37}$ (SNU)	Kamiokande ( $10^6 \text{ cm}^{-2}\text{s}^{-1}$ )	$\text{Ga}^{71}$ (SNU)
Experimental result	$2.55 \pm 0.25$	$2.75 \pm 0.45$	$73.7 \pm 7.6$
Princeton/Yale model	$8 \pm 1$	$5.7 \pm 0.8$	$132 \pm 6$
Saclay model	$6.4 \pm 1.4$	$4.4 \pm 1.1$	$123 \pm 7$

### 3.3 Solar neutrinos

Solar neutrinos have been known for some time to deviate from expectations. The experimental results [16, 17, 18, 19], coming from three different types of detectors, as well as the predictions of the Princeton-Yale [20] and Saclay [21] solar models are summarized in table 2. With the only exception of  $\text{Ga}^{71}$ , experimental results are dominated by systematic errors. Furthermore, it can be seen that theoretical uncertainties almost always exceed experimental errors. Thus, in any comparison between models and experiments, theoretical errors must necessarily be taken into account. As they are highly correlated, we have limited our choice of theoretical models to those published and for which the complete error correlation matrix is available [22].

With neutrino mass differences as large as those of eqs. (8) and (10), all solar neutrino oscillation-induced phenomena observable on Earth are bound to be energy-independent. In particular, in all experiments the  $\nu_e$  flux must be reduced by the same factor  $F$ , independently of the detection threshold.

Although marginally in some cases, the data are consistent with this expectation. Fig. 1 shows the two  $\chi^2$ 's obtained by comparing the experimental data with the theoretical predictions of the Princeton-Yale (PY) and Saclay (S) solar models as a function of  $F$ . In the analytical formulation of these  $\chi^2$ 's, theoretical and experimental errors have been added in quadrature and correlations among theoretical errors have been fully taken into account by means of the appropriate correlation matrices. The  $\chi^2_{\text{PY}}$  and  $\chi^2_{\text{S}}$  minima turn out to be  $\chi^2_{\text{PY}_{\min}} = 9.48$  (CL = 0.9%, equivalent to 2.6 sigma) and  $\chi^2_{\text{S}_{\min}} = 5.10$  (CL = 7.8%, equivalent to 1.8 sigma). The confidence level of the first fit is admittedly rather low, but not unacceptable. In view also of the goodness of the second fit and of the only approximate knowledge of both theoretical and experimental errors (particularly with regard to their Gaussian-like behaviour), no conclusion about a compelling energy-dependence of the  $F$ -factor can be drawn at this stage.

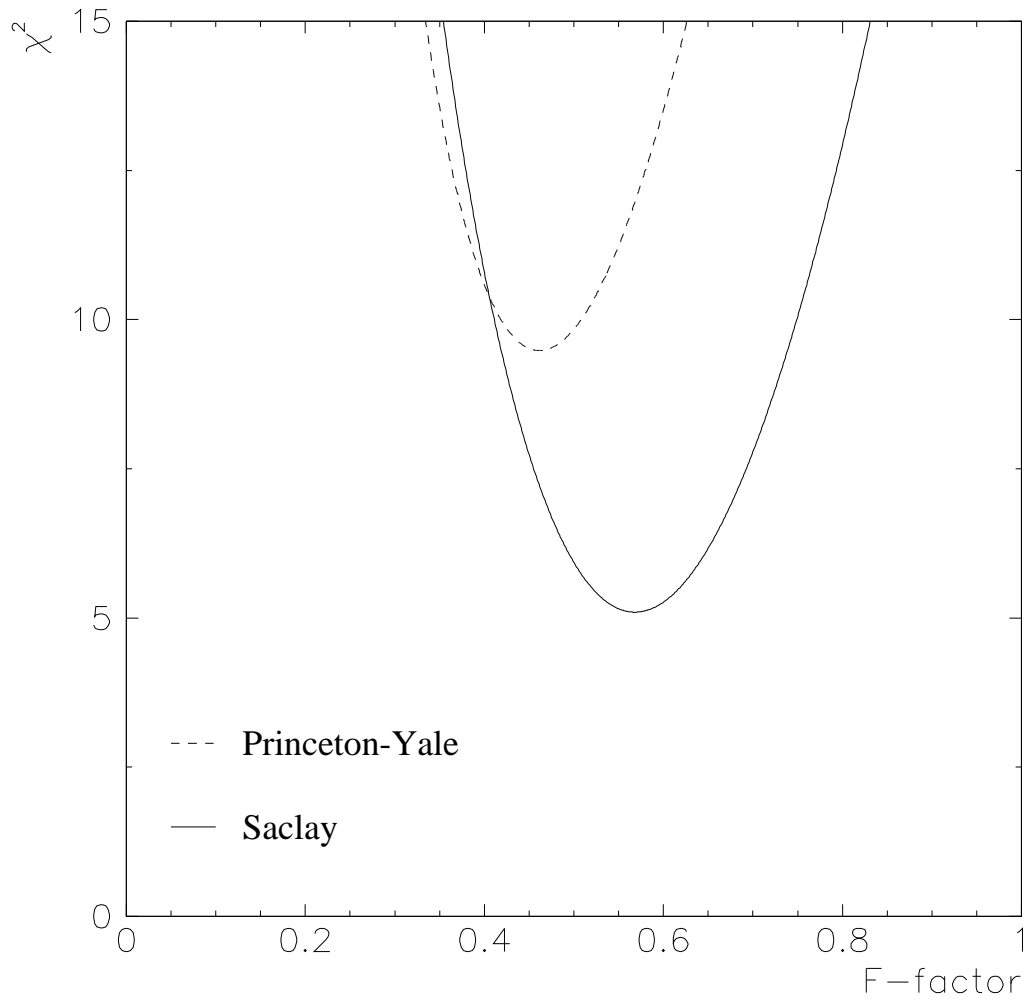


Figure 1: The functions  $\chi^2 = \chi^2(F)$  obtained by comparing the experimental data with the predictions of the Princeton-Yale (dashed line) and Saclay (full line) solar models (see Table 2).  $F$  is the energy-independent factor by which all  $\nu_e$  fluxes predicted by the same model are reduced. For each model the correlations among errors on the expected rates for the various detectors have been taken into account through the use of the appropriate error correlation matrix. All errors have been assumed to be Gaussian-distributed.

Following the procedure normally used by the Particle Data Group [5], the errors on the  $F$ -factors have been multiplied by the appropriate scale-factors. Then, the two  $F$ -factors turn out to be  $F_{\text{PY}} = 0.46 \pm 0.13$  and  $F_{\text{S}} = 0.57 \pm 0.13$ , implying the existence of a solar neutrino deficit.

In order to quantify this deficit for our further analysis, we have calculated a new  $F$ -factor taking the arithmetic average of  $F_{\text{PY}}$  and  $F_{\text{S}}$ . We have taken the error on this average ( $\pm 0.08$ ) to be a good estimator of a systematic error reflecting the fact that the two models of the same Sun yield different predictions. We have added this error in quadrature to the typical statistical error ( $\pm 0.13$ ), arriving at the result

$$F = 0.51 \pm 0.15. \quad (12)$$

Using the formalism of the three-flavour analysis, eq. (12) trivially translates into

$$P_{ee}^L = 0.51 \pm 0.15. \quad (13)$$

## 4 Mass eigenstates and mixing matrix

Taking  $m_1 \ll m_2$ , eqs. (8) and (10) yield

$$m_1 \ll 10^{-2} \text{ eV}, \quad m_2 = (0.18 \pm 0.06) \text{ eV}, \quad m_3 = (19.4 \pm 0.7) \text{ eV} \quad (14)$$

These values are consistent with the predictions of the see-saw model [23]. Using a leptonic quadratic hierarchy and the above value of  $m_3$  one has  $m_1 = 1.6 \times 10^{-6} \text{ eV}$  and  $m_2 = 6.8 \times 10^{-2} \text{ eV}$ .

From the three eqs. (7), (11) and (13), within the constraints (4), the three angles  $\theta_{ij}$  are uniquely determined to be

$$\theta_{12} = 0.55 \pm 0.08, \quad \theta_{13} = 0.38 \pm 0.06, \quad \theta_{23} < 0.03. \quad (15)$$

with  $\chi_{\min}^2 = 0.093$ .

As in any solution with  $m_3$  larger than a few eV, the smallness of  $\theta_{23}$  is dictated by the upper limits (4) and this implies an over-determination of the angles  $\theta_{12}$  and  $\theta_{13}$ . The good consistency of the input data is illustrated in fig. 2, which shows the three relations between  $\theta_{12}$  and  $\theta_{13}$  obtained from eqs. (7), (11) and (13) for  $\theta_{23} = 0$ . It can be seen, for instance, that the value of  $\theta_{13}$  is determined not just by the result of eq. (7) but also by the system formed by the other two.

The  $U$ -matrix is then

$$U = \begin{pmatrix} 0.79 \pm 0.05 & 0.49 \pm 0.06 & 0.37 \pm 0.05 \\ -0.52 \pm 0.06 & 0.85 \pm 0.05 & < 0.03 \\ -0.31 \pm 0.04 & -0.20 \pm 0.05 & 0.93 \pm 0.03 \end{pmatrix} \quad (16)$$

The knowledge of the angles  $\theta_{ij}$  allows also to calculate the following short- and long-baseline average transition probabilities  $P_{\alpha\beta}^S$  and  $P_{\alpha\beta}^L$

$$\begin{array}{lll} P_{ee}^S = 0.77 \pm 0.06 & P_{e\mu}^S < 3 \times 10^{-4} & P_{e\tau}^S = 0.23 \pm 0.06 \\ & P_{\mu\mu}^S > 0.998 & P_{\mu\tau}^S < 1.5 \times 10^{-3} \\ & & P_{\tau\tau}^S = 0.77 \pm 0.06 \end{array}$$

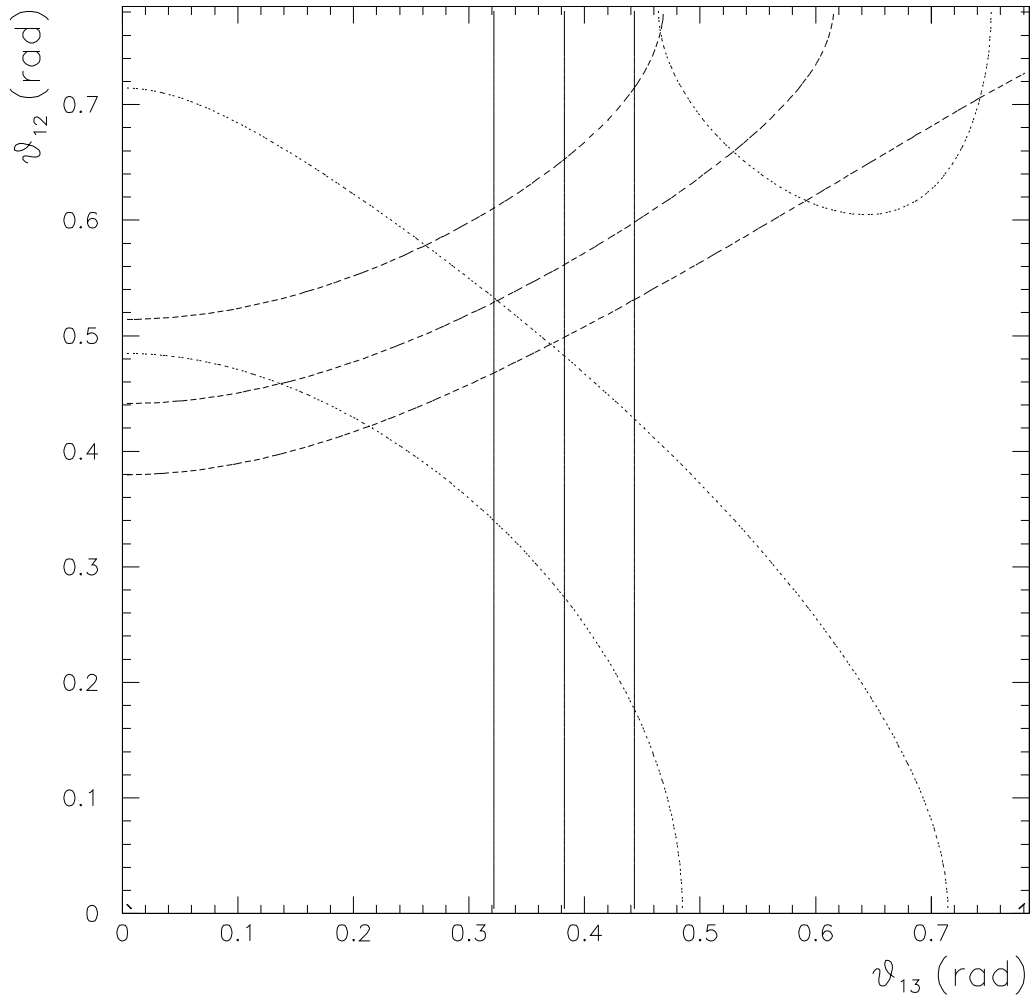


Figure 2: The three relations ( $\pm 1\sigma$ ) between  $\theta_{12}$  and  $\theta_{13}$  obtained from eqs. (7) (full lines), (11) (dashed lines) and (13) (dotted lines) for  $\theta_{23} = 0$ .



$$\begin{array}{lll}
P_{ee}^L = 0.47 \pm 0.06 & P_{e\mu}^L = 0.35 \pm 0.05 & P_{e\tau}^L = 0.19 \pm 0.05 \\
& P_{\mu\mu}^L = 0.60 \pm 0.06 & P_{\mu\tau}^L = 0.05 \pm 0.03 \\
& & P_{\tau\tau}^L = 0.76 \pm 0.06
\end{array}$$

All upper limits are at the 90% CL.

## 5 Discussion

The results for the mixing angles (eq. (15)) are that  $\theta_{12}$  is fairly sizeable,  $\theta_{23}$  is small and  $\theta_{13}$  lies somewhere in between the two. Small values of  $\theta_{23}$  are typical of any solution in which the mass of the heaviest eigenstate (eq. (14)) is of cosmological relevance [24].

The main implications of this scenario are discussed below.

### 5.1 Accelerators

Short-baseline experiments like Chorus and Nomad [25] are expected to see a  $\nu_\tau$  signal. If not from  $\nu_\mu$ - $\nu_\tau$  ( $P_{\mu\tau}^S$  is consistent with zero), at least from  $\nu_e$ - $\nu_\tau$  transitions (owing to the about 1%  $\nu_e$  component in the beam and the sizeable  $P_{e\tau}^S$ ).

Long-baseline experiments [7, 26] have a better chance to detect  $\nu_\mu$ - $\nu_\tau$  oscillations, but the largest effect is anticipated in the  $\nu_e$ - $\nu_\mu$  channel.

Experiments at low energy accelerators like KARMEN [27] and LSND [28] ought to be able to detect  $\nu_e$ - $\nu_\mu$  transitions as, owing to the onset of the slow oscillation,  $P_{e\mu}$  in their accepted range of  $L/E$  is typically a fraction of a percent. Both experiments should be in the position of investigating the conservation of the  $\nu_e$  flux, a study which so far has resulted in the KARMEN not yet statistically significant  $P_{ee}^S$  lower limit [27].

### 5.2 Beta decay and reactors

For Majorana neutrinos, under very reasonable assumptions, the neutrino-less double beta decay amplitude is proportional to [29]

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i \quad (17)$$

The value of  $m_{\beta\beta}$  calculated from eqs. (14) and (16) is 2.6 eV, while its present upper limit is only about several eV [5], so that experiments will soon be in the position of providing definite conclusions on this very important issue.

In about 13% of all  $\beta$ -decays a heavy neutrino of mass  $m_3$  (eq.(14)) is expected to be produced. Two-state analyses of the spectra end-points [30] might be able to investigate this question. However, at the present time, the presence of ununderstood phenomena in the high energy region precludes any firm conclusions [31].

All  $\beta$ -sources are expected to yield lower-than-canonically-calculated  $\nu_e$  fluxes (both  $P_{ee}^S$  and  $P_{ee}^L$  are smaller than 1 and the wave-length of the fast oscillation is about 5 mm/MeV) but the only result available so far [32] is not accurate enough to really test this possibility.

This effect should also be present in experiments at nuclear reactors. After some initial tantalizing results indicating a large depletion of the  $\nu_e$  flux at the two sigma level [33, 34], the more recent experiments have failed to substantiate any deviation from expectations. In fact, their claims represent the most stringent limit to oscillations of the  $\nu_e$  and the most serious challenge to the result of eq. (5).

For mass differences as large as that of eq. (6), owing to the very short wave-length of the oscillation, these experiments consist in comparing the antineutrino flux expected from the reactor with that experimentally measured at some (short-baseline) distance away. In spite of the many such experiments carried out in the last decade [35, 36, 37, 38, 39], no significant progress has been made in the accuracy of the results. The comparison of ref. [35] with ref. [39] (the two best documented and, respectively, the oldest and newest papers) shows that in both cases the determination of the ratio between results and expectations is limited by a systematic error of about 6%. This error is largely dominated by a common uncertainty on the knowledge of the reactor antineutrino energy spectrum. This is due to the fact that the bulk of the data on thermal neutron fission induced  $\beta$ -spectra [40, 41, 42] all reactor antineutrino energy spectra calculations are based on were obtained only once, more than a decade ago. It has been argued repeatedly that such important measurements ought to be checked, that the procedure used in deriving neutrino spectra from  $\beta$ -spectra ought to be better understood, that the quoted systematic uncertainty is probably underestimated and that the over-all normalization error in the reactor antineutrino energy spectrum is more likely to be around 10% [43, 44, 45]. In any case, even taking the quoted 6% at face value and assuming it to be Gaussian-distributed, the reactor result differs from the value of  $P_{ee}^S$  resulting from eq. (15) by 2.7 sigma, indicating some discrepancy but not a really serious incompatibility.

Reactor experiments are potentially sensitive also to the existence of the mass difference of eq. (10). In this case the wavelength of the oscillation is certainly long enough to allow a comparison between two measurements of the antineutrino flux in two detectors at different distances from the reactor, thus reducing the systematic effects discussed above. Because of the low event rate, experiments so far have been barely able to investigate the  $L/E$  region of interest. However, the claimed limits [35, 36, 37, 38, 39] exclude in part or almost completely the region of eq. (10). On the other hand, even if in a different context, the re-analysis of the reactor data of ref. [46] has shown that a mass  $m = (0.085 \pm 0.010)$  eV is quite consistent with all existing evidence. Reactor experiments with an even slightly larger  $L$  than it has been possible so far are clearly needed to clarify this important point. Long-base experiments will also have the benefit of a projected larger depletion of the  $\nu_e$  flux ( $P_{ee}^L < P_{ee}^S$ ).

In conclusion, although reactor experiments do indicate the existence of some potential problems at the very edges of the explored region in the  $\sin^2(2\alpha), \delta m^2$  plane, the large uncertainties involved preclude any firm conclusion at the present time.

### 5.3 Astrophysics and cosmology

Neutrinos with masses of the order of ten eV can be the dark matter particles needed to explain the high rotation velocities in the outer parts of spiral galaxies [47, 48] and the observed X-ray emission from hot diffused gas in elliptical galaxies [49].

Heavy neutrinos lead also to some important cosmological consequences. With masses as large as those of eq. (14), neutrinos contribute far more than baryons to the total matter density of the universe  $\Omega_0 = \rho_0/\rho_c$  where  $\rho_c$  is the critical density (the subscript 0 indicates present-day values).

In terms of the Hubble parameter  $H_0$  ( $H_0 = 100 h_0 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ) the baryon density  $\Omega_b$  is known to be [5]

$$\Omega_b h_0^2 = (0.015 \pm 0.005) \quad (18)$$

while the neutrino density  $\Omega_\nu$  obtained from eq. (14) is

$$\Omega_\nu h_0^2 = 1.075 \times 10^{-2} \times \sum_i m_i = (0.211 \pm 0.008). \quad (19)$$

Thus, for a universe whose only massive stable components are baryons and neutrinos, the total density is given by

$$\Omega_0 h_0^2 = (\Omega_b + \Omega_\nu) h_0^2 = (0.226 \pm 0.009) \quad (20)$$

The relation resulting from eq. (20) is shown in the  $\Omega_0, H_0$  plane of Fig. 3 by the full-line curves. They define the allowed region ( $1\sigma$ ) for a universe composed only of baryons (visible matter) and neutrinos (hot dark matter, HDM). The allowed region for a possible third massive component (cold dark matter, CDM) lies on the right-hand side of these curves. The same figure shows also the relations between  $\Omega_0$  and  $H_0$  obtained from the Einstein's equations for the two limiting values of the age of the universe  $T = 13, 16$  Gyr and for two different values of the cosmological constant  $\Lambda$  ( $\Lambda = 3H_0^2\lambda_0/c^2$ )  $\lambda_0 = 0, 1$ .

To date, the value of  $H_0$  is not precisely known, the various experimental determinations ranging between the values  $h_0 \simeq 0.5$  and  $h_0 \simeq 0.8$  [50].

Depending on the value of  $H_0$ , different situations can occur.

1. For  $h_0 \simeq 0.5$  the choice  $\lambda_0 = 0$  is consistent with the accepted bounds on the age of the universe. In this case eq. (20) requires  $\Omega_0 \gtrsim 0.85$  and for  $\Omega_0 = 1$  neutrinos turn out to be the most massive component of the

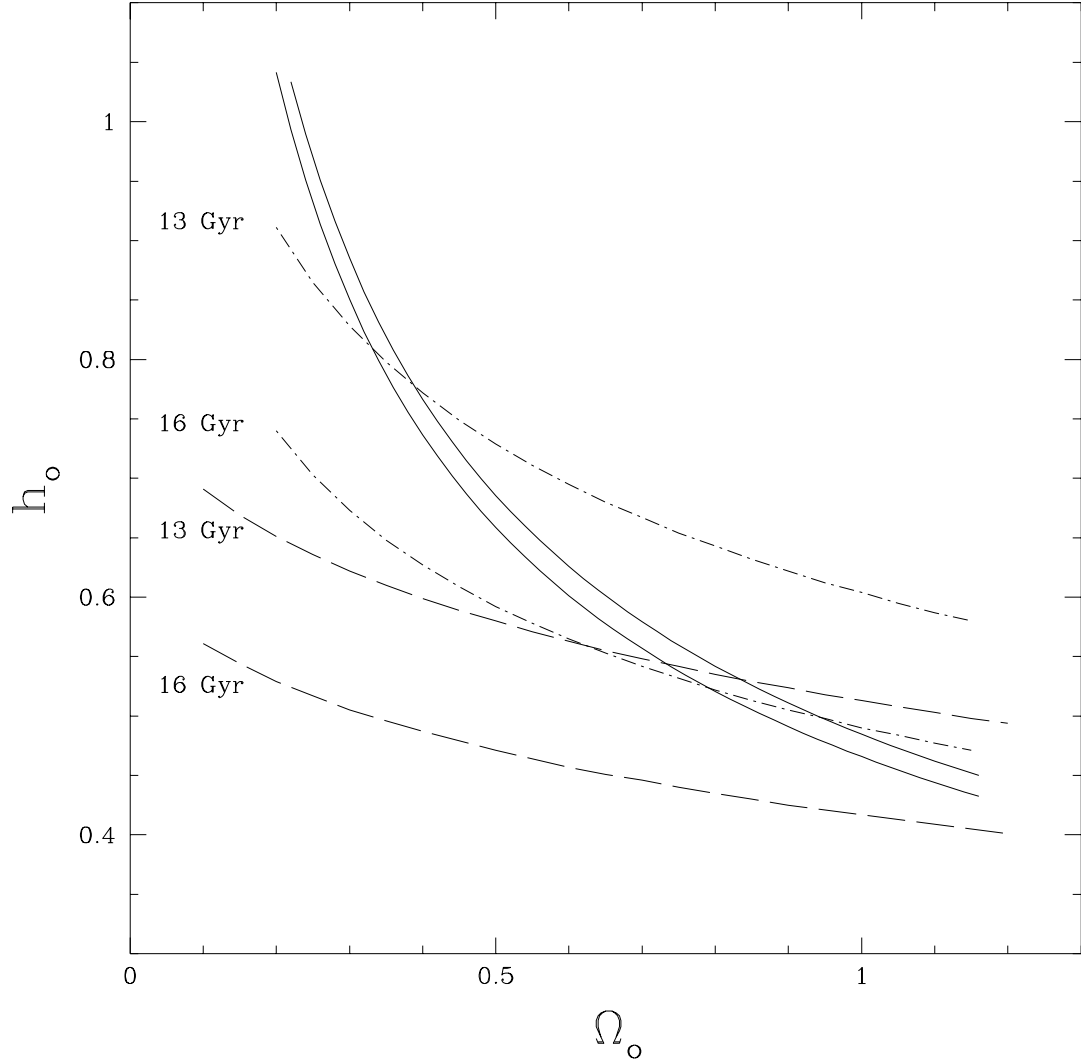


Figure 3: Relations between the total matter density of the universe  $\Omega_0$  (in units of critical density) and the normalized Hubble parameter  $h_0$  ( $H_0 = 100 h_0 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ). The full lines are for a sum of the neutrino masses of  $(19.6 \pm 0.7) \text{ eV}$ . The other curves represent the function  $h_0 = h_0(\Omega_0)$  for the two limiting values of the age of the universe  $T = 13 \text{ Gyr}$  and  $T = 16 \text{ Gyr}$  and for values of the scaled cosmological constant  $\lambda_0$  ( $\lambda_0 = \Lambda c^2 / 3H_0^2$ )  $\lambda_0 = 0$  (dashed lines) and  $\lambda_0 = 1$  (dotted-dashed lines).

universe. For only baryons and neutrinos contributing to the total density of a critical universe, the result of eq. (20) implies

$$\begin{aligned} H_0 &= (47.4 \pm 0.9) \text{ km s}^{-1} \text{ Mpc}^{-1} \\ T &= (13.8 \pm 0.3) \text{ Gyr.} \end{aligned} \tag{21}$$

2. For  $h_0 \simeq 0.65$ , eq. (20) requires  $\lambda_0 > 0$ . For  $\lambda_0 = 1$ ,  $\Omega_0 \gtrsim 0.5$  and the presence of cold dark matter could be accomodated with a ratio  $\Omega_{CDM}/\Omega_{HDM}$  ( $\Omega_{CDM}/\Omega_\nu$ ) which, depending on the value of  $\Omega_0$ , could be as large as  $\approx 0.5$ .
3. For  $h_0 \simeq 0.8$  and  $\lambda_0 = 1$ ,  $\Omega_0 \simeq 0.4$  and the mass of the universe is again due solely to baryons and neutrinos. Non-vanishing values of  $\Omega_{CDM}/\Omega_{HDM}$  are possible only for  $\lambda_0 > 1$ .

Thus, for any reasonable values of  $H_0$  and  $\lambda_0$ , neutrinos with masses as large as those of eq. (14) are the dominant component of the mass of the universe. As a consequence, their existence contradicts all cold-dark-matter-dominated (CHDM) cosmological models [51]. To have the same success that these models have in explaining the present data on the power spectrum of density fluctuations, hot-dark-matter-dominated models will have to rely on some new hypotheses, such as, for instance, seeds of non-Gaussian origin [52, 53].

## 6 Conclusions

This analysis shows that, at least taking results at face value, the evidence for the existence of neutrino oscillations is really substantial. Although none of the existing positive signals is admittedly beyond some criticism, their ensemble is quite compelling.

The signature of neutrino oscillations lies in the  $\sin^2$  factor of eq. (1) and any convincing proof of their existence must rely on some experimental observation of an  $L/E$  dependence. This evidence, first obtained from the analysis of the  $L/E$  modulation of the  $\nu_\mu$ - $\nu_e$  asymmetry in prompt neutrinos [4, 6], has been further strengthened recently by the observation of the dependence on the zenith-angle (effectively also  $L/E$ ) of the ratio between the  $\nu_\mu$  and  $\nu_e$  fluxes in atmospheric neutrinos of multi-GeV energy [13].

All the available experimental information from accelerator, atmospheric and solar neutrinos is accounted for in the framework of a three-flavour neutrino oscillation analysis. The solution found is unique and not significantly contradicted by any existing result, all conflicting evidence being below the three sigma level.

Neutrino physics is presently at a crossroads. If  $m_3$  is indeed large enough to be of cosmological relevance, the limits of eq. (4) imply that  $\theta_{23}$  is very small. The relations between  $\theta_{12}$  and  $\theta_{13}$  are then those of fig. 2 and all indications are that  $\theta_{13}$  is sizeable. Under this condition, high energy accelerator experiments

such as Chorus and Nomad should not fail to see a  $\nu_\tau$  signal. If they do, this implies that at least some of the input data to our analysis are wrong and that either  $\theta_{13}$  or  $m_3$  or both are smaller than the results we have arrived at. In the first case, the atmospheric neutrino result (see fig. 2) implies the relative large value for the  $F$ -factor  $F = 0.70 \pm 0.07$ , thus disfavouring solar models such as that of Princeton-Yale [20] which (see fig. 1) require  $F \lesssim 0.5$ . In the other two cases  $m_3$  must be smaller than a few eV, thus excluding neutrinos as the main constituents of the dark matter of the universe and leaving them out of a primary role in astrophysics.

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